# Risk, Expected Return, and Expected Utility

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Uncertainty is something which everyone must deal with every day of their lives. The imperfections of our world impose on us, the necessity to make decisions without certainty of outcome. The question arises then, as to how investors make logical and rational decisions? This essay attempts to examine the nature of the uncertainty faced by an investor, and the possible approaches that he/she might take to deal with it.

Section one sets the context by defining risk and uncertainty. Section two then considers the investor's response to these phenomena. The concept of expected utility is here introduced. Finally, section three looks beyond expected utility in noting some alternatives theories. It is concluded that, although it exhibits a number of inconsistencies, the expected utility maximization principle remains relatively robust.

**Risk and uncertainty**

Rutterford (1983) mentions several types of risk associated with investment, ranging from uncertainty of default to interest rate risk. The terms "risk" and "uncertainty" appear to be used interchangeably, but for the purposes of this discourse, a distinction is drawn between them. As Bacharach (1976) and others have pointed out, risk is measurable, while uncertainty is not.

Risk, in essence, is quantifiable uncertainty. Inherent in the concept of risk is the assumption that an individual can formulate - either subjectively or objectively - a probability distribution for various outcomes. Without such a distribution (a situation of uncertainty), it is not possible to accurately and consistently reflect the risk elements in the return on investments. Christy & Clendenin (1978) proposed the idea that what is popularly called risk, is actually uncertainty. It is arguable, however, that, in investment analysis, it must be feasible to assess in some way the probabilities of various outcomes. Hence, what is commonly called uncertainty is actually risk.

Even if the market is rational and risk is fully reflected in returns, it may not be possible to predict all possible outcomes, let alone their relative probabilities. How then can investors make informed investment decisions. Shackle (1955) suggested that investors concentrate on "focus values". These focus values (one favourable and one unfavourable) represent a summary of the possible outcomes. He suggests that people examine only focus values when making a decision. While this may seem a little simplistic, it can form a reasonable solution to the investment problem. Agents only consider the most likely outcomes in their analysis, secure in the knowledge that the omission of minor outcomes will not unduly affect their optimal decision. Despite the fact that Keynes (1936) believed uncertainty to be immeasurable (c.f. Aiginger,1987), it can be contended that investors do (at least subjectively) estimate risk.

**Dealing with risk**

Having arrived at some interpretation of future outcomes, how should the rational

investor behave? The profit-maximizing investor will pursue the strategy that maximizes expected returns. However, this theory, while intuitively satisfactory, does not hold in practice. Wu and Zakon (1972) state that the hypothesis of expected returns must be rejected on the evidence of the widespread existence of diversified portfolios. A more widely noted indictment is the "St. Petersberg Paradox" (see Luce and Faiffa, 1957). This was initially developed by Bernoulli.

Consider a game involving the flicking of a coin, such that the prize for partaking equalled £$2^x$, where x is the number of heads thrown before a tail. How much would one be willing to pay to play the game? Under the principle of insufficient reason (c.f. Savage, 1972), one would assess the probability of a head on each flip as 0.5, and arrive at an infinite expected return. Yet, realistically, people would not be prepared to pay extremely high prices to play such a game.

Shackle's (1958) idea of focus values does provide one possible solution to this problem. More conventional answers rely on utility theory (Levy and Sarnet, 1972). It is argued that investors are concerned with utility as opposed to monetary values. Cramer and Bernoulli forwarded square-root and logarithmic utility functions as plausible alternatives, yet as Luce and Faiffa (1957) point out, while these furnish solutions to the St. Petersberg problem, they are confounded by other paradoxes.

The concept of utility is still, however, useful. Von Neumann & Morgenstern set about proving the expected utility maximization criterion by use of a series of axioms. Unfortunately, their theory is virtually impossible to verify empirically. Attempts at the cardinal measurement of utility have been made, but with little success. Yet as Varian (1987) and others note, utility may be measured ordinally, and agents can reveal preferences without having to state how many cardinal utils they get from each outcome. Observation of such revealed preferences indicates that agents do tend to act rationally. From this, it is possible to infer that investors formulate consistent strategies that we can expect them to follow.

Expected utility theory, while useful, is also very general. This generality means that it is very difficult to explicitly disprove. Rather than attempt to do so, the following section will outline some possible extensions and alternatives.

**Beyond expected utility**

One simple extension is that provided by Corner & Mayes (1983). They note that investors are averse to downside risk. Under such "safety first" principles, it is argued that the skewness coefficient, as well as the variance, should be noted in assessing the risk element of an investment. Expected utility can provide for this reaction only by giving higher weights to the disutility of losses than to the utility of gains .

Aiginger (1987) points to two alternatives to expected utility theory. The first of these is known as "prospect theory", which originated in the works of Kahneman and Tversky. It was developed in response to the fact that some empirically revealed preferences differ from those predicted by the expected utility hypothesis. The first anomaly is the "certainty effect", or the underweighing of outcomes by investors in certain conditions. The second is the "isolation effect", involving a disregard for certain components common to all prospects. The final inconsistency is the "reflection effect", which arises when people exhibit risk-averseness to gains but not to losses. Prospect theory sees decisions as being based on both gains and losses rather than just losses. It replaces probabilities with "decision weights". However, while it represents an improvement on expected utility theory, in

no way does it offer a complete panacea.

The second alternative that Aiginger (1987) mentions is called "regret theory". This was also developed in response to empirical inconsistencies. It suggests that one must account for the sensations of regret or rejoicing which follow from making a decision under conditions of uncertainty. Once again, however, such a thesis is extremely difficult to empirically verify. In the last resort, as Aiginger (1987) points out: "...the expected utility maximization criterion is the only hypothesis that is clearly both convincing and operational."

## Conclusion

In this essay, the importance of risk evaluation to the investor was discussed. Investors must be able to make a statement about the uncertainties which they face in order to be able to enter the market. A number of theories, including expected return maximization, and expected utility theory, can be invoked to explain their behaviour, once they have assigned probabilities to outcomes. In conclusion it can be argued that a rational investor's behaviour will conform to the expected utility maximization criterion, because it accounts for preferences and individuality within a logical framework, something which alternative theories fail to do.

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